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LETTER TO THE EDITOR

Reply to *Comment on "Development of an Improved Gas-Kinetic BGK Scheme for Inviscid and Viscous Flows*"

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In the *comment on* "*Development of an Improved Gas-Kinetic BGK Scheme for Inviscid and Viscous Flows*" by Xu [7], it is argued that the modification of the BGK scheme proposed in Ref. [1] neglects a few critically important terms which are essential for the accuracy of the Navier–Stokes solutions.

The integral solution of the BGK model used in Ref. [1] is written as

$$\begin{aligned} f(0,0,t,u,v,\xi) = {} & \left(1-e^{-t/\tau}\right)g_0 + e^{-t/\tau} f_0(-ut,-vt) \\ & + \tau\left(-1+e^{-t/\tau}\right)(u\bar{a}+v\bar{b})g_0 \\ & + te^{-t/\tau}(u\bar{a}+v\bar{b})g_0. \end{aligned} \tag{1}$$

It is different from

$$\begin{aligned} f(0,0,t,u,v,\xi) = {} & \left(1-e^{-t/\tau}\right)g_0 + e^{-t/\tau} f_0(-ut,-vt) \\ & + \tau\left(-1+e^{-t/\tau}\right)(u\bar{a}+v\bar{b}+A)g_0 \\ & + te^{-t/\tau}(u\bar{a}+v\bar{b})g_0 + tAg_0, \end{aligned} \tag{2}$$

which is used as a basis of the original BGK scheme in Ref. [1]. The only difference between the two formulations is the treatment related to the time evolution term A.

We believe that the neglected terms from the time slope A should not cause any trouble in Navier–Stokes calculations and that the modified scheme loses none of the key features of the BGK model. The main role of the numerical flux from A is to couple the spatial slopes of the BGK model in such a way that the resultant numerical flux is similar to the Lax–Wendroff scheme with second order temporal accuracy, as was analyzed in Refs. [2, 3]. The reason for neglecting the time slope A is purely to improve computational efficiency

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and convergence. First, in order to incorporate the effect of A completely, a substantial increase in computational time is unavoidable since it involves expensive matrix algebra. Even though the time evolution flux is fully considered, its impact on solution accuracy does not seem to be critical, at least in *steady state* aerodynamic problems, which were the focus in Ref. [1]. From the numerical point of view, the role of A is to provide a time integration term based on the evolution physics of the BGK model. Thus, it may be replaced by another more efficient time integration strategy for steady state calculations so long as it does not compromise solution accuracy. With the complete form of the BGK flux, it looks very difficult to achieve the high level of computational efficiency required of modern numerical schemes.

Second, the rate of convergence of the original BGK flux has to be improved substantially in order to compute steady state problems efficiently. The main reason is the numerical flux related to the temporal slope A, which couples spatial slopes, through the conservation requirement, produce a flux like the Lax–Wendroff scheme. Although this property is very desirable for unsteady computations, it unfortunately makes the convergence to a steady state solution very slow. Thus, the proper modification of A is necessary in one way or another.

Since we pointed out a possible problem in the original BGK scheme that might arise in calculating compressible Navier-Stokes flows [1], we ran the code based on the original BGK scheme downloaded from the website at www.mmath.ust.hk/~makxu. The code seems to be nearly the same as our in-house code used in Ref. [1] except that the former still keeps the time slope term, non-dimensionalized differently, and has second order spatial accuracy at the wall. In a flat plate laminar boundary layer problem, the u velocity profiles are fine, as shown in Fig. 1, while the v velocity profile shows a slight deviation from the exact solution. These results are somewhat different from the ones that were presented in Ref. [1], which are also based on the original BGK scheme. The source of the difference between the computed results remains to be further investigated. In a shock boundary layer test case, the downloaded code was modified to have numerical flux at the wall with either first or second order spatial accuracy, since our in-house codes usually treat the numerical flux at the wall with first order accuracy. The skin friction coefficients for each boundary treatment are

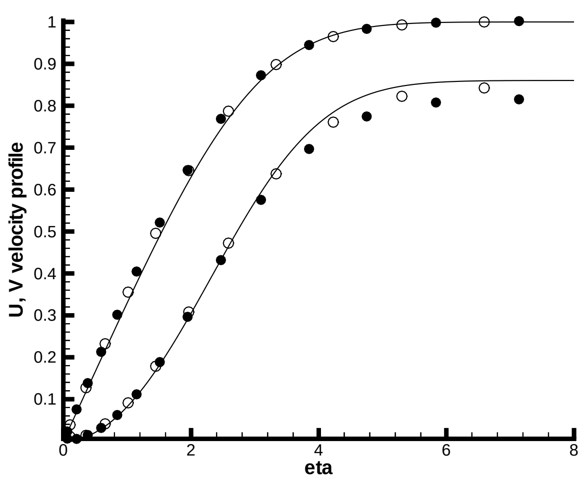

FIG. 1. U, V velocity profiles of flat plate at the location $x = 6.438$ (circle) and $x = 34.469$ (filled circle). Solid lines indicate exact solutions. Free stream conditions are $Re = 10^5$ and $M = 0.3$.

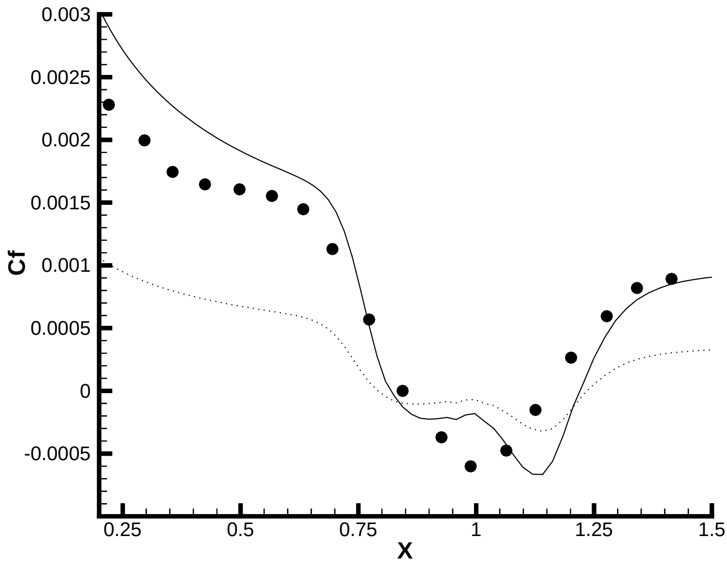

FIG. 2. Skin friction coefficient along the plate. $M = 2.0$ and $Re = 2.96 \times 10^5$. Solid line indicates the result from the second order at wall. Dotted line from the first order at wall. Circles denote the experimental result from Ref. [6].

shown in Fig. 2. The result with first order accuracy at the wall deviates significantly from the experimental result, while the one with second order yields good agreement. However, scheme 2 of Ref. [1] produces accurate results even with first order accuracy at the wall (see Fig. 14 in Ref. [1]). This indicates that the crucial factor which influences solution accuracy is related to the numerical flux at the wall rather than the treatment of the time slope A. This also justifies the modification of f_0 to develop scheme 2 in Ref. [1].

In conclusion, the original BGK scheme in Ref. [5] might have difficulties in steady state compressible Navier–Stokes computations due to computational inefficiency, slow convergence, and the f_0 term, which may prevent the exact capture of contact discontinuity. Similar difficulties might also arise if the CFL number is reduced so that the EFM flux [6] from f_0 becomes more dominant in the BGK numerical flux. Therefore, an improved numerical treatment of the time slope A and the initial distribution function f_0 is necessary without losing the key feature of the BGK model, which was one of the main topics in Ref. [1].

REFERENCES

1. D. Chae, C. Kim, and O. H. Rho, Development of an improved gas-kinetic BGK scheme for inviscid and viscous flows, *J. Comput. Phys.* **158**, 1 (2000).
2. K. Xu, L. Martinelli, and A. Jameson, Gas-kinetic finite volume methods, flux vector splitting, and artificial diffusions, *J. Comput. Phys.* **120**, 48 (1995).
3. C. Kim and A. Jameson, A robust and accurate LED–BGK solver on unstructured adaptive meshes, *J. Comput. Phys.* **143**, 598 (1998).
4. G. Degrez, C. H. Boccardoro, and J. F. W. Wendt, The interaction of an oblique shock wave with a laminar boundary layer revisited. An experimental and numerical study, *J. Fluid Mech.* **177**, 246 (1987).
5. K. Xu and K. H. Prendergast, Numerical Navier–Stokes solutions from gas-kinetic theory, *J. Comput. Phys.* **114**, 9 (1994).
6. D. Pullin, Direct simulation methods for compressible inviscid ideal-gas flow, *J. Comput. Phys.* **43**, 231 (1980).
7. K. Xu, Comment on "Development of an improved gas-kinetic BGK scheme for inviscid and viscous flows," *J. Comput. Phys.*